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DESIGN CONSIDERATIONS FOR WINGS HAVING MINIMUM DRAG DUE TO LIFT

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SUMMARY

The problem of increasing the range of supersonic aircraft by the use of twisted and cambered wings is considered, primarily for the purpose of developing a rational method for the selection of a design lift coefficient. Relations and curves are presented from which a suitable selection may be made, depending on the relative importance of maximum range and top speed.

INTRODUCTION

The inherently low values of lift-drag ratio and the resultant short ranges characteristic of conventional aircraft configurations operated at supersonic speeds have stimulated research on more efficient shapes for use at these speeds. This research has of necessity been directed along two main lines, one the reduction of the drag at zero lift and the other the reduction of the drag due to lift. Along the latter line, one of the most promising developments has been the use of twist and camber to reduce the wing drag due to lift for a given plan form. Contributions to this field have been made by Robert T. Jones (refs. 1 to 3), E. W. Graham and his coworkers (refs. 4 and 5), and S. H. Tsien (ref. 6), among others.

One of the principal problems with which the reports just mentioned are concerned may be stated thus: Given a wing plan form operating at a given lift and a given Mach number, find the shape (as expressed by the angle of attack, the twist, and the camber) which will result in the lowest drag. Attention is directed to the words "at a given lift and a given Mach number" since the optimum shape will change both with the lift and with the Mach number. For any particular aircraft, however, practical considerations will usually dictate the use of a wing having fixed twist and camber, even though the aircraft may be required to operate over a range of lift coefficient as well as a range of Mach

number. Out of this situation arises the problem of how best to select the one particular shape that will represent the best compromise for all flight conditions, which amounts to selecting the lift coefficient at which the drag is to be minimized at a given Mach number. The present paper offers some suggestions concerning this selection process.

SYMBOLS

C_D drag coefficient, $\dfrac{\text{Drag}}{\text{Dynamic pressure} \times \text{Reference area}}$

$C_{D_{0,f}}$ drag coefficient of flat wing at zero lift

$C_{D_{0,w}}$ drag coefficient of particular minimum-drag wing at zero lift

C_{D_d} drag coefficient of particular minimum-drag wing at design lift coefficient

$C_{D_{m,w}}$ lowest drag coefficient for a particular minimum-drag wing

ΔC_D drag coefficient measured from the flat-wing zero-lift drag coefficient, $C_D - C_{D_{0,f}}$ (used with the various subscripts)

C_L lift coefficient, $\dfrac{\text{Lift}}{\text{Dynamic pressure} \times \text{Reference area}}$

C_{L_d} design lift coefficient

C_{L_e} lift coefficient at which drag of particular minimum-drag wing equals drag of flat wing

$C_{L_{m,w}}$ lift coefficient at which $C_{D_{m,w}}$ is obtained

$C_{L_{opt,f}}$ lift coefficient at which $(L/D)_{max,f}$ is obtained

$C_{L_{opt,w}}$ lift coefficient at which $(L/D)_{max,w}$ is obtained

$C_{L_{opt,w}}'$ lift coefficient at which $\left(L^{1/2}/D\right)_{max,w}$ is obtained

$(L/D)_{max,f}$	maximum lift-drag ratio for flat wing
$\left(L^{1/2}/D\right)_{max,f}$	maximum $(\text{lift})^{1/2}$-drag ratio for flat wing
$(L/D)_{max,w}$	maximum lift-drag ratio for particular minimum-drag wing
$\left(L^{1/2}/D\right)_{max,w}$	maximum $(\text{lift})^{1/2}$-drag ratio for particular minimum-drag wing
K_f	drag-rise factor for flat wing, $\partial C_D/\partial {C_L}^2$
K_w	drag-rise factor for family of minimum-drag wings, $\partial C_D/\partial {C_L}^2$
$\beta = \sqrt{M^2 - 1}$	
M	Mach number
m	cotangent of sweepback angle of leading edge
$C_{L_\alpha} = \partial C_L/\partial\alpha$	
α	angle of attack

Superscripts:

*	condition of maximum $\dfrac{(L/D)_{max,w}}{(L/D)_{max,f}}$
**	condition of maximum $\dfrac{\left(L^{1/2}/D\right)_{max,w}}{\left(L^{1/2}/D\right)_{max,f}}$

ANALYSIS AND DISCUSSION

Factors Influencing the Range

It is anticipated that aircraft operating at supersonic speeds will usually be powered by a reaction propulsion system (turbojet, ram jet, or rocket, for example), for which the fuel rate (weight of fuel used per unit time) will be closely proportional to the thrust, rather than to the horsepower, as in the case of a propeller-driven airplane. For such aircraft, the maximum range will be attained by flying at the condition of $(L/D)_{max}$ if the flight Mach number is specified, or at the condition of $\left(L^{1/2}/D\right)_{max}$ if the ratio of the atmospheric pressure to the wing loading is specified (specifying this ratio is roughly equivalent to specifying the altitude). These relations are illustrated in figure 1.

The characteristics of the propulsion system enter into the preceding statements only to the extent that the fuel rate is assumed to be proportional to the thrust (for example, the changes in available thrust with changes in altitude or Mach number do not affect the considerations). In the case of any specific aircraft, however, the engine characteristics will be a primary factor in the determination of the actual operating point for maximum range. Although a detailed discussion of this problem is not the purpose of this paper, it is mentioned so as to emphasize the fact that the operating point for a particular aircraft may be neither at $(L/D)_{max}$ nor at $\left(L^{1/2}/D\right)_{max}$, but at some point between these two conditions. Therefore, after the main assumptions and relations have been stated, the following analysis is divided into two main parts, one dealing with $(L/D)_{max}$ and the other dealing with $\left(L^{1/2}/D\right)_{max}$, so that the limits of probable design conditions are covered.

Basic Assumptions and Relations

The drag coefficient of the flat wing is assumed to be expressible as a parabolic function of the lift coefficient for a given Mach number and wing plan form:

$$C_D = C_{D_{O,f}} + K_f C_L^2 \tag{1}$$

This equation is plotted in figure 2. As long as the relation between C_D and C_L is parabolic, the curve can be regarded as applying either to a complete aircraft configuration or to an isolated wing. For convenience, however, the term "flat wing" is used herein when referring to the curve. The drag of the family of wings which at any value of C_L has the lowest possible drag is assumed to be known (from the work of ref. 6, for example, or from some other suitable source) and to be given by the following equation:

$$C_D = C_{D_{0,f}} + K_w {C_L}^2 \tag{2}$$

This equation is also plotted in figure 2.

The first step is to write the equation for the drag of one of the family of minimum-drag wings, that is, of a wing which has a fixed twist and camber. Now the addition of small amounts of twist and camber to a flat wing introduces increments of drag and lift which do not vary with angle of attack. The drag of a wing with fixed twist and camber can therefore be represented by the following equation:

$$C_D = C_{D_{m,w}} + K_f\left(C_L - C_{L_{m,w}}\right)^2 \tag{3}$$

where $C_{D_{m,w}}$ and $C_{L_{m,w}}$ are shown in figure 2. If equation (3) is to express the drag of one of the family of minimum-drag wings, the value of C_D given by equation (3) must first be equated to the value given by equation (2) for some particular lift coefficient C_{L_d}, which is called the design lift coefficient. In addition, since the drag given by equation (2) is the lowest possible at any value of C_L, then the curves represented by equations (2) and (3) must be made tangent at $C_L = C_{L_d}$. These two conditions serve to determine $C_{D_{m,w}}$ and $C_{L_{m,w}}$ in equation (3), and the equation for the drag of the particular minimum-drag wing becomes

$$C_D = C_{D_{0,f}} + \left(1 - \frac{K_w}{K_f}\right)K_w {C_{L_d}}^2 + K_f\left[C_L - \left(1 - \frac{K_w}{K_f}\right)C_{L_d}\right]^2 \tag{4}$$

The relations just discussed are illustrated in figure 2. The discussion of the curve represented by equation (4), in particular its relation to equation (1), forms the remainder of the paper. Some miscellaneous relations for the various coefficients are presented in appendix A.

Design Considerations Based on $(L/D)_{max}$

Maximum lift-drag ratio.- As mentioned previously, twist and camber may be used to produce greater values of the lift-drag ratio, and in particular of the maximum lift-drag ratio $(L/D)_{max,w}$, than can be obtained from the flat wing. From equations (4) and (1) the ratio of the maximum lift-drag ratio for any particular minimum-drag wing $(L/D)_{max,w}$ to the corresponding value for the flat wing $(L/D)_{max,f}$ can be obtained as a function of the two quantities K_w/K_f and $C_{L_d}/C_{L_{opt,f}}$, where $C_{L_{opt,f}}$ is the lift coefficient corresponding to $(L/D)_{max,f}$. The ratio is as follows:

$$\frac{(L/D)_{max,w}}{(L/D)_{max,f}} = \frac{1}{\sqrt{1 + \left(1 - \frac{K_w}{K_f}\right)\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2} - \left(1 - \frac{K_w}{K_f}\right)\frac{C_{L_d}}{C_{L_{opt,f}}}} \tag{5}$$

For the flat wing the following relations hold:

$$\left.\begin{aligned} (L/D)_{max,f} &= \frac{1}{2\sqrt{K_f C_{D_{0,f}}}} \\ C_{L_{opt,f}} &= \sqrt{\frac{C_{D_{0,f}}}{K_f}} \end{aligned}\right\} \tag{6}$$

The ratio given by equation (5) is plotted in figure 3(a). Of some practical interest is the fact that for a given value of K_w/K_f (which amounts to a given plan form and Mach number) there is a particular design lift coefficient at which the greatest increase in $(L/D)_{max}$ is obtained. If an asterisk is used to denote the condition of maximum $\frac{(L/D)_{max,w}}{(L/D)_{max,f}}$, then the following relation can be written:

$$\left[\frac{(L/D)_{max,w}}{(L/D)_{max,f}}\right]^* = \frac{C_{L_d}}{C_{L_{opt,f}}} = \frac{1}{\sqrt{K_w/K_f}} \tag{7}$$

Equation (7) is shown as the dashed line in figure 3(a). One criterion is thus provided for the selection of C_{L_d}, if the only item of concern in the particular design under consideration is the maximum possible increase in $(L/D)_{max}$, to the exclusion of other items (such as increases in minimum drag).

The numerical values of the $(L/D)_{max}$ ratio in figure 3(a) are also of interest. As shown in appendix B, values of K_w/K_f in the neighborhood of 0.5 or 0.6 can probably be expected for reasonable plan forms and Mach numbers. From figure 3(a), increases in $(L/D)_{max}$ of 30 to 40 percent can therefore probably be realized by the introduction of the proper twist and camber.

Other characteristics.- In connection with the lift-drag ratio, the lift coefficient corresponding to $(L/D)_{max}$ is of importance. The ratio of $C_{L_{opt,w}}$ to $C_{L_{opt,f}}$ is given by the following equation:

$$\frac{C_{L_{opt,w}}}{C_{L_{opt,f}}} = \sqrt{1 + \left(1 - \frac{K_w}{K_f}\right)\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2} \tag{8}$$

This ratio is, of course, always greater than unity.

In the case of configurations for which the maximum speed capabilities, as well as the maximum range, are important, the increase in minimum drag caused by the addition of twist and camber to the flat wing must be considered. (Although a detailed study should include the effect of Mach number, the change in minimum drag with Mach number is small enough so that for the purposes of this paper the constant Mach number case can be considered.) This increase in minimum drag can be expressed as follows:

$$\frac{C_{D_{m,w}}}{C_{D_{O,f}}} = 1 + \frac{K_w}{K_f}\left(1 - \frac{K_w}{K_f}\right)\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2 \tag{9}$$

and the lift coefficient at which the minimum drag occurs can be determined from

$$\frac{C_{L_{m,w}}}{C_{L_{opt,f}}} = \left(1 - \frac{K_w}{K_f}\right)\frac{C_{L_d}}{C_{L_{opt,f}}} \tag{10}$$

If, as before, an asterisk is used to denote the condition of maximum $\frac{(L/D)_{max,w}}{(L/D)_{max,f}}$, then equations (8) to (10) become

$$\left(\frac{C_{L_{opt,w}}}{C_{L_{opt,f}}}\right)^* = \frac{C_{L_d}}{C_{L_{opt,f}}} = \frac{1}{\sqrt{K_w/K_f}} \tag{11a}$$

$$\left(\frac{C_{D_{m,w}}}{C_{D_{0,f}}}\right)^* = 2 - \frac{1}{\left(C_{L_d}/C_{L_{opt,f}}\right)^2} = 2 - \frac{K_w}{K_f} \tag{11b}$$

(The reason for the seemingly strange result given by this equation for the impractical (or ideal) case of $K_w \longrightarrow 0$ can be understood if a plot is made similar to fig. 3(c) but with $C_{L_d}/C_{L_{opt,w}}$ as the abscissa. The case of $K_f \longrightarrow \infty$ can be visualized easily by making a sketch similar to fig. 2.)

$$\left(\frac{C_{L_{m,w}}}{C_{L_{opt,f}}}\right)^* = \frac{C_{L_d}}{C_{L_{opt,f}}} - \frac{1}{C_{L_d}/C_{L_{opt,f}}} = \frac{1 - \frac{K_w}{K_f}}{\sqrt{K_w/K_f}} \tag{11c}$$

Equations (8) to (11) are plotted in figures 3(b) to 3(d). It is instructive to compare the maximum increase in $(L/D)_{max}$ with the corresponding increase in minimum drag, that is, the values given by the dashed lines of figures 3(a) and 3(c), which come from equations (7) and (11b). These values are plotted in figure 4 and show that for a realistic range of K_w/K_f (greater than 0.4) the percentage increase in minimum drag is always greater than that in $(L/D)_{max}$, if the design condition is chosen to give the greatest possible increase in $(L/D)_{max}$. This situation is very undesirable, of course, if the top speed of the configuration under study is important, since the top speed is greatly dependent on the minimum drag.

Evidently if the top speed of the configuration is important, a more satisfactory design condition would be one which would yield something less than the maximum possible $(L/D)_{max}$ increase but which would keep

the minimum-drag increase within acceptable limits. Such a design condition is possible, as shown by the data presented in figure 5. As an example, consider a curve with $K_w/K_f = 0.5$. The maximum possible $(L/D)_{max}$ increase of 41 percent is achieved only at the expense of a 50-percent increase in minimum drag. However, if the design condition is modified so that a (say) 30-percent increase in $(L/D)_{max}$ is obtained (by choosing a value of $C_{L_d}/C_{L_{opt,f}}$ of about 0.68), then the minimum-drag penalty is decreased to 12 percent. Similar considerations apply, of course, to any of the other curves with K_w/K_f constant. As indicated in the example just discussed, the lines of constant $C_{L_d}/C_{L_{opt,f}}$ give the proper value of design lift coefficient to use once a satisfactory design condition, as measured by the gain in $(L/D)_{max}$ balanced against the penalty in minimum drag, is selected.

Design Considerations Based on $\left(L^{1/2}/D\right)_{max}$

Considerations analogous to those developed in the preceding section entitled "Design Considerations Based on $(L/D)_{max}$" can be expressed on the basis of $\left(L^{1/2}/D\right)_{max}$. This development proceeds in a manner similar to that for the case of $(L/D)_{max}$.

<u>Maximum value of $L^{1/2}/D$</u>.- The equation for $\left(L^{1/2}/D\right)_{max}$ corresponding to equation (5) is

$$\frac{\left(L^{1/2}/D\right)_{max,w}}{\left(L^{1/2}/D\right)_{max,f}} = \frac{\left(C_L{}^{1/2}/C_D\right)_{max,w}}{\left(C_L{}^{1/2}/C_D\right)_{max,f}}$$

$$= \frac{3^{3/4}\sqrt{\left(1 - \frac{K_w}{K_f}\right)\frac{C_{L_d}}{C_{L_{opt,f}}} + \sqrt{3 + \left(4 - \frac{K_w}{K_f}\right)\left(1 - \frac{K_w}{K_f}\right)\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2}}}{3 + 3\left(1 - \frac{K_w}{K_f}\right)\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2 - \left(1 - \frac{K_w}{K_f}\right)^2\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2 - \left(1 - \frac{K_w}{K_f}\right)\frac{C_{L_d}}{C_{L_{opt,f}}}\sqrt{3 + \left(4 - \frac{K_w}{K_f}\right)\left(1 - \frac{K_w}{K_f}\right)\left(\frac{C_{L_d}}{C_{L_{opt,f}}}\right)^2}} \tag{12}$$

For the flat wing, the following relations hold:

$$\left.\begin{aligned}
\left(\frac{C_L{}^{1/2}}{C_D}\right)_{max,f} &= \frac{3^{3/4}}{4K_f{}^{1/4}C_{D_{0,f}}{}^{3/4}} = \frac{3^{3/4}}{4K_fC_{L_{opt,f}}{}^{3/2}} \\
C_{L_{opt,f}} &= \sqrt{\frac{C_{D_{0,f}}}{K_f}}
\end{aligned}\right\} \tag{13}$$

It is emphasized that $C_{L_{opt,f}}$ has the same meaning here that it had in the previous development; that is, it is the lift coefficient corresponding to $(L/D)_{max,f}$ $\left(\text{not } \left(L^{1/2}/D\right)_{max,f}\right)$.

The ratio given by equation (12) is plotted in figure 6(a). If a double asterisk is used to denote the condition of maximum $\frac{(L^{1/2}/D)_{max,w}}{(L^{1/2}/D)_{max,f}}$, then the following relation holds:

$$\left[\frac{\left(L^{1/2}/D\right)_{max,w}}{\left(L^{1/2}/D\right)_{max,f}}\right]^{**} = 3^{1/4}\sqrt{\frac{C_{L_d}}{C_{L_{opt,f}}}} = \frac{1}{\left(K_w/K_f\right)^{1/4}} = \sqrt{\left[\frac{(L/D)_{max,w}}{(L/D)_{max,f}}\right]^{*}} \tag{14}$$

Equation (14) is plotted as the dashed line in figure 6(a) and provides a criterion for the selection of C_{L_d}, the basis of selection in this case being the development of the maximum possible increase in $\left(L^{1/2}/D\right)_{max}$, to the exclusion of other factors.

<u>Other characteristics</u>.- The lift coefficient corresponding to $\left(L^{1/2}/D\right)_{max,w}$ is designated herein by $C_{L'_{opt,w}}$. The ratio of $C_{L'_{opt,w}}$ to $C_{L_{opt,f}}$ is given by the following equation, which corresponds to equation (8) for the case of $(L/D)_{max}$:

$$\frac{C_{L'_{opt,w}}}{C_{L_{opt,f}}} = \frac{1}{3}\frac{C_{L_d}}{C_{L_{opt,f}}}\left[1 - \frac{K_w}{K_f} + \sqrt{\frac{3}{\left(C_{L_d}/C_{L_{opt,f}}\right)^2} + \left(4 - \frac{K_w}{K_f}\right)\left(1 - \frac{K_w}{K_f}\right)}\right] \tag{15}$$

The equations for minimum drag and the lift coefficient at which minimum drag occurs (eqs. (9) and (10)) apply to the present case as well as to the previous one.

If, as in equation (14), a double asterisk is used to indicate the condition of maximum $\frac{(L^{1/2}/D)_{max,w}}{(L^{1/2}/D)_{max,f}}$, then equations (15), (9), and (10) take the following forms:

$$\left(\frac{C_{L'_{opt,w}}}{C_{L_{opt,f}}}\right)^{**} = \frac{C_{L_d}}{C_{L_{opt,f}}} = \frac{1}{\sqrt{3K_w/K_f}} = \frac{1}{\sqrt{3}}\left(\frac{C_{L_{opt,w}}}{C_{L_{opt,f}}}\right)^{*} \tag{16a}$$

$$\left(\frac{C_{D_{m,w}}}{C_{D_{0,f}}}\right)^{**} = \frac{4}{3} - \frac{1}{9\left(C_{L_d}/C_{L_{opt,f}}\right)^2} = \frac{1}{3}\left(4 - \frac{K_w}{K_f}\right) \tag{16b}$$

$$\left(\frac{C_{L_{m,w}}}{C_{L_{opt,f}}}\right)^{**} = \frac{C_{L_d}}{C_{L_{opt,f}}} - \frac{1}{3C_{L_d}/C_{L_{opt,f}}} = \frac{1 - \frac{K_w}{K_f}}{\sqrt{3K_w/K_f}} = \frac{1}{\sqrt{3}}\left(\frac{C_{L_{m,w}}}{C_{L_{opt,f}}}\right)^{*} \tag{16c}$$

Equations (9), (10), (15), and (16) are plotted in figures 6(b) to 6(d). A comparison of the maximum increase in $\left(L^{1/2}/D\right)_{max}$ with the corresponding increase in C_{D_m} is given in figure 7. The values in figure 7 come from equations (14) and (16b). The comparison is rather more favorable than the corresponding comparison in figure 4, but in some cases the balance between $\left(L^{1/2}/D\right)_{max,w}$ and $C_{D_{m,w}}$ represented by figure 7 may be inacceptable. In such cases a more suitable design compromise can be selected from the curves of figure 8, which corresponds to figure 5 for the $(L/D)_{max}$ case. Considerations similar to those discussed in connection with figure 5 apply to figure 8.

Comparison of Designs Based on $(L/D)_{max}$ and on $\left(L^{1/2}/D\right)_{max}$

Because of the existence of at least two design bases (that is, $(L/D)_{max}$ and $\left(L^{1/2}/D\right)_{max}$) the question naturally arises as to what extent a design which emphasizes one of these factors is penalized with respect to the other. Some indication may be had from the answer to the following specific question and its converse: If $C_{L_d}/C_{L_{opt,f}}$ is chosen to give $\left[\frac{(L/D)_{max,w}}{(L/D)_{max,f}}\right]^{*}$, what percentage of $\left[\frac{(L^{1/2}/D)_{max,w}}{(L^{1/2}/D)_{max,f}}\right]^{**}$ will be

realized? The answers to these questions are shown in figure 9, together with the minimum-drag increase and the design lift coefficient for each design condition. The first point to be made from the figure is that, regardless of which factor $\left(\text{that is, } (L/D)_{max} \text{ or } \left(L^{1/2}/D\right)_{max}\right)$ is maximized, the increase in the other is always at least 92 percent of the maximum possible increase. The second point is that the minimum-drag increase resulting from maximizing $\left(L^{1/2}/D\right)_{max}$ is much less than that resulting from maximizing $(L/D)_{max}$. Therefore, a design lift coefficient chosen to give the maximum possible increase in $\left(L^{1/2}/D\right)_{max}$ will result in a wing which will develop at least 93 percent of the maximum possible increase in $(L/D)_{max}$ but which has a much smaller minimum drag than if the design had been chosen to give the maximum possible increase in $(L/D)_{max}$. Such a design lift coefficient therefore suggests itself as a good choice as long as the minimum drag is not too critical. For cases in which closer attention must be paid to the minimum drag, a more suitable compromise can be selected from figures 5 and 8.

CONCLUDING REMARKS

Under the assumption that, for a given wing plan form, the lowest drag obtainable by the use of twist and camber is known, information has been presented which will allow the rational selection of a design lift coefficient. It is suggested that, for a given Mach number and wing plan form, a design lift coefficient which gives the largest possible increase in the maximum $(\text{lift})^{1/2}$-drag ratio is a reasonable choice as long as the minimum drag is not too critical. If the increase in minimum drag due to twist and camber must be kept very small, a somewhat lower design lift coefficient should be used.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 8, 1954.

APPENDIX A

MISCELLANEOUS RELATIONS FOR THE VARIOUS COEFFICIENTS

Certain relations, although not directly bearing on the main purpose of the paper, are nevertheless of incidental interest and are easily derived. For example, if the values of C_D given by equations (1) and (4) are equated and the resultant lift coefficient is called C_{L_e} (see fig. 2), the following simple result is obtained:

$$\frac{C_{L_e}}{C_{L_d}} = \frac{1}{2} \tag{A1}$$

Other similar relations can be obtained. For example, if ΔC_D is used to denote the drag coefficient measured from the flat-wing zero-lift drag coefficient as a reference, that is,

$$\Delta C_D = C_D - C_{D_{O,f}} \tag{A2}$$

then these relations are as follows (see fig. 2 for the physical meaning of the various coefficients):

$$\left.\begin{aligned}
\frac{C_{L_{m,w}}}{C_{L_d}} &= \frac{\Delta C_{D_{m,w}}}{\Delta C_{D_d}} = 1 - \frac{K_w}{K_f} \\
\frac{\Delta C_{D_{m,w}}}{\Delta C_{D_{O,w}}} &= \frac{K_w}{K_f} \\
\frac{\Delta C_{D_{O,w}}}{\Delta C_{D_d}} &= \left(1 - \frac{K_w}{K_f}\right)\Bigg/\frac{K_w}{K_f}
\end{aligned}\right\} \tag{A3}$$

The preceding relations are shown in figure 10.

APPENDIX B

PROBABLE RANGE OF K_w/K_f

In reference 2, Robert T. Jones gave an equation for the minimum drag (at a fixed lift) of a slender wing lying near the center of the Mach cone. This equation was also given later by Adams and Sears in reference 7, with some discussion of the derivation. For wings with zero tip chord, the equation may be written in the notation of the present paper as follows:

$$\frac{1}{\beta}K_w = \frac{1}{A/m}\,\frac{1 + 2(\beta m)^2}{\pi\beta m} \tag{B1}$$

The drag for the corresponding flat wing, if the leading edge is assumed to develop no thrust, is simply

$$\frac{1}{\beta}K_f = \frac{1}{\beta C_{L_\alpha}} \tag{B2}$$

so that

$$\frac{K_w}{K_f} = \frac{\beta C_{L_\alpha}}{A/m}\,\frac{1 + 2(\beta m)^2}{\pi\beta m} \tag{B3}$$

Values of βC_{L_α} can be obtained from various sources, such as reference 8.

Values of K_w/K_f calculated from equation (B3) are plotted in figure 11. Also included are some more exact values for the conical triangular wing ($A/m = 4$), taken from reference 6. The agreement between these values and those calculated from equation (B3) is good. Although equation (B1) is not strictly applicable to a wing with $A/m = 6$, the values in figure 11 are included to show the trend of the variation of K_w/K_f with A/m. The lower values of K_w/K_f obtained for the higher values of A/m indicate that the use of proper twist and camber should be more beneficial for arrow-type plan forms than for diamond-type plan forms. Values of K_w/K_f in the neighborhood of 0.5 or 0.6 can probably be expected for reasonable plan forms and Mach numbers.

As mentioned previously, the values of K_w/K_f presented in figure 11 are based on the assumption that no leading-edge thrust is developed in the flat wing. For increasing amounts of leading-edge thrust, the values will move closer to unity.

REFERENCES

1. Jones, Robert T.: Estimated Lift-Drag Ratios at Supersonic Speed. NACA TN 1350, 1947.

2. Jones, Robert T.: The Minimum Drag of Thin Wings in Frictionless Flow. Jour. Aero. Sci., vol. 18, no. 2, Feb. 1951, pp. 75-81.

3. Jones, Robert T.: Theoretical Determination of the Minimum Drag of Airfoils at Supersonic Speeds. Jour. Aero. Sci., vol. 19, no. 12, Dec. 1952, pp. 813-822.

4. Graham, E. W.: A Drag Reduction Method for Wings of Fixed Plan Form. Jour. Aero. Sci., vol. 19, no. 12, Dec. 1952, pp. 823-825.

5. Rodriguez, A. M., Lagerstrom, P. A., and Graham, E. W.: Theorems Concerning the Drag Reduction of Wings of Fixed Plan Form. Jour. Aero. Sci., vol. 21, no. 1, Jan. 1954, pp. 1-7.

6. Tsien, S. H.: The Supersonic Conical Wing of Minimum Drag. Ph. D. Thesis, Cornell Univ., June 1953.

7. Adams, Mac C., and Sears, W. R.: Slender-Body Theory – Review and Extension. Jour. Aero. Sci., vol. 20, no. 2, Feb. 1953, pp. 85-98.

8. Piland, Robert O.: Summary of the Theoretical Lift, Damping-in-Roll, and Center-of-Pressure Characteristics of Various Wing Plan Forms at Supersonic Speeds. NACA TN 1977, 1949.

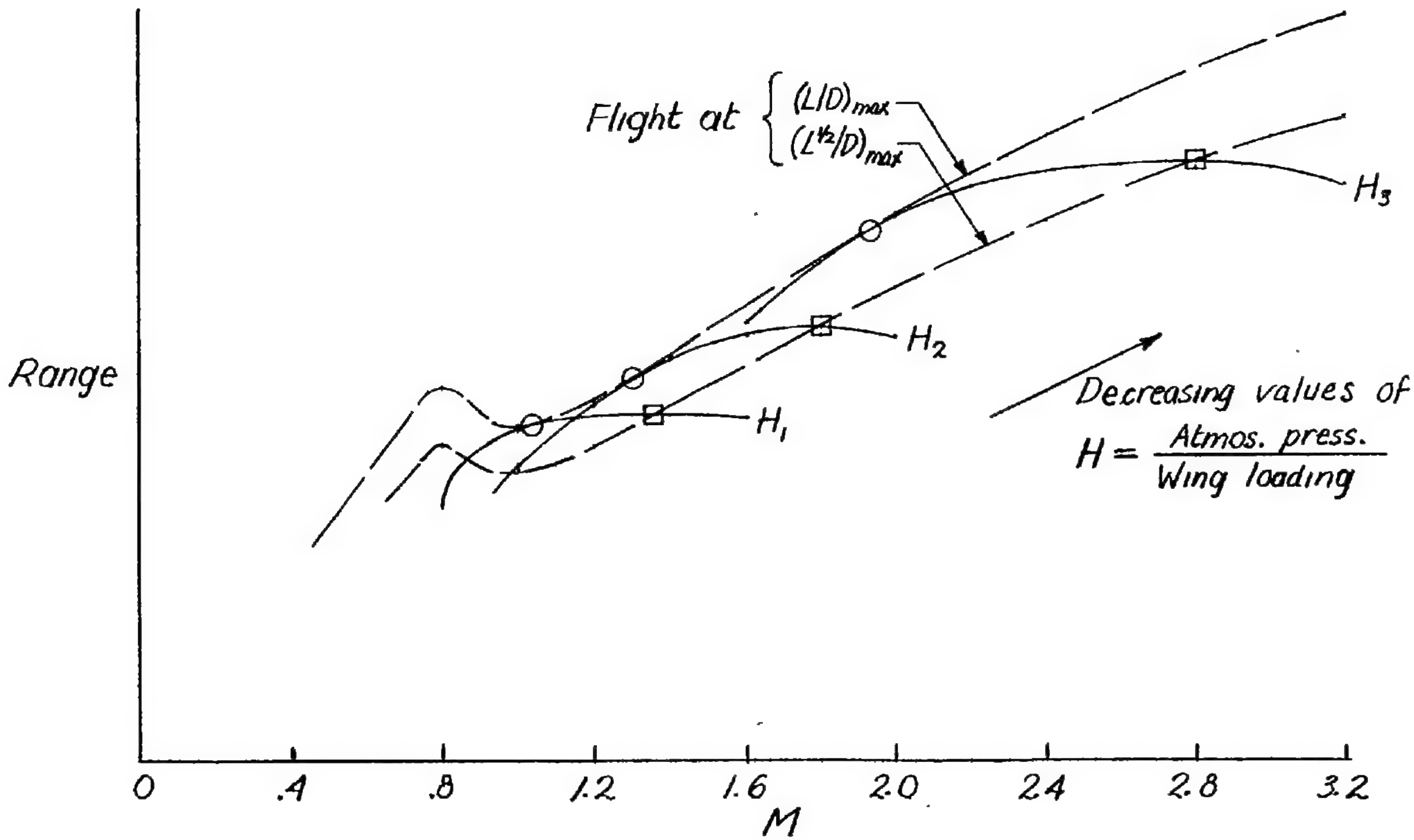


Figure 1.- Range relations for a typical set of aerodynamic characteristics. Fuel rate proportional to thrust (turbojet, for example).

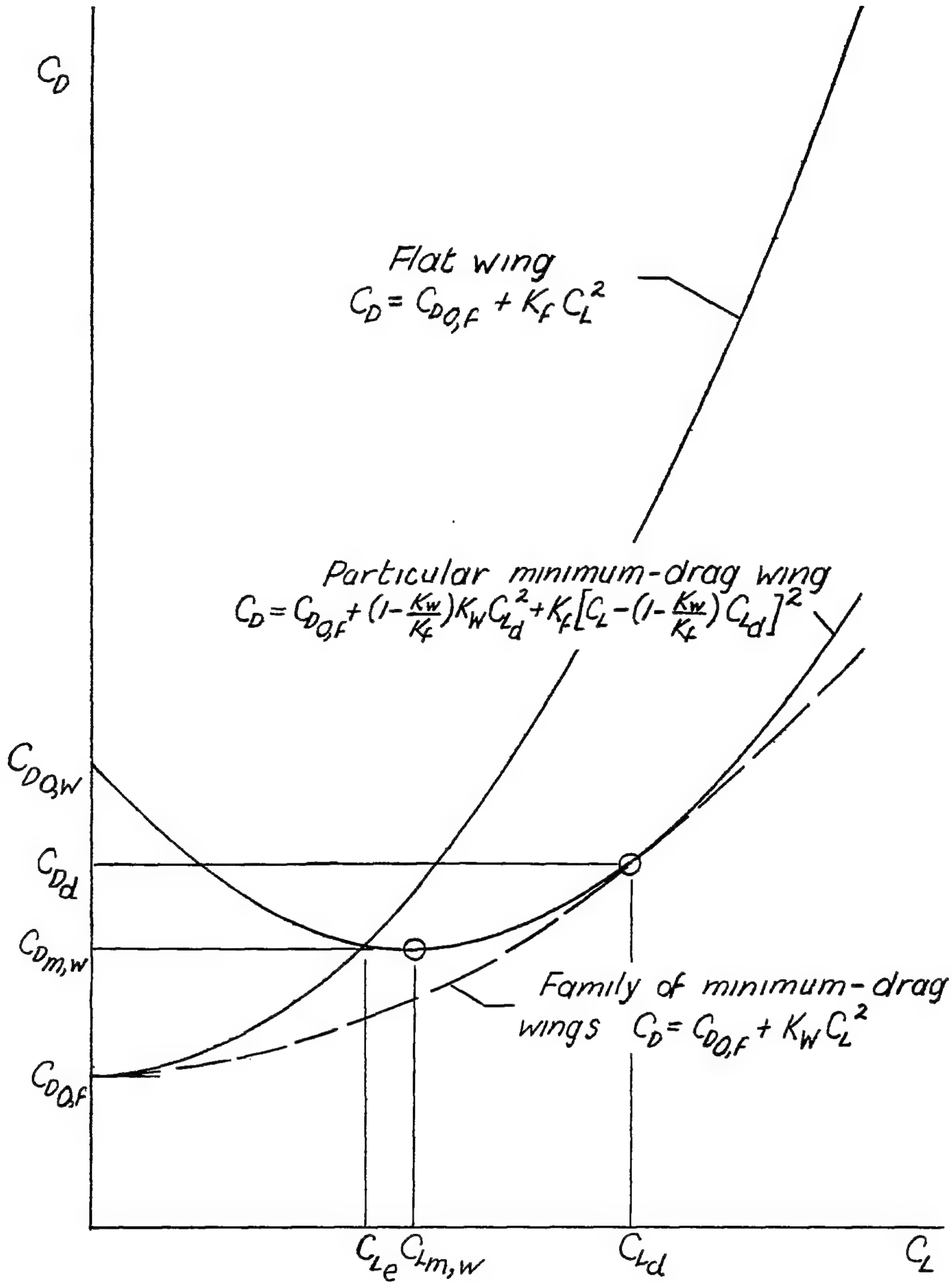
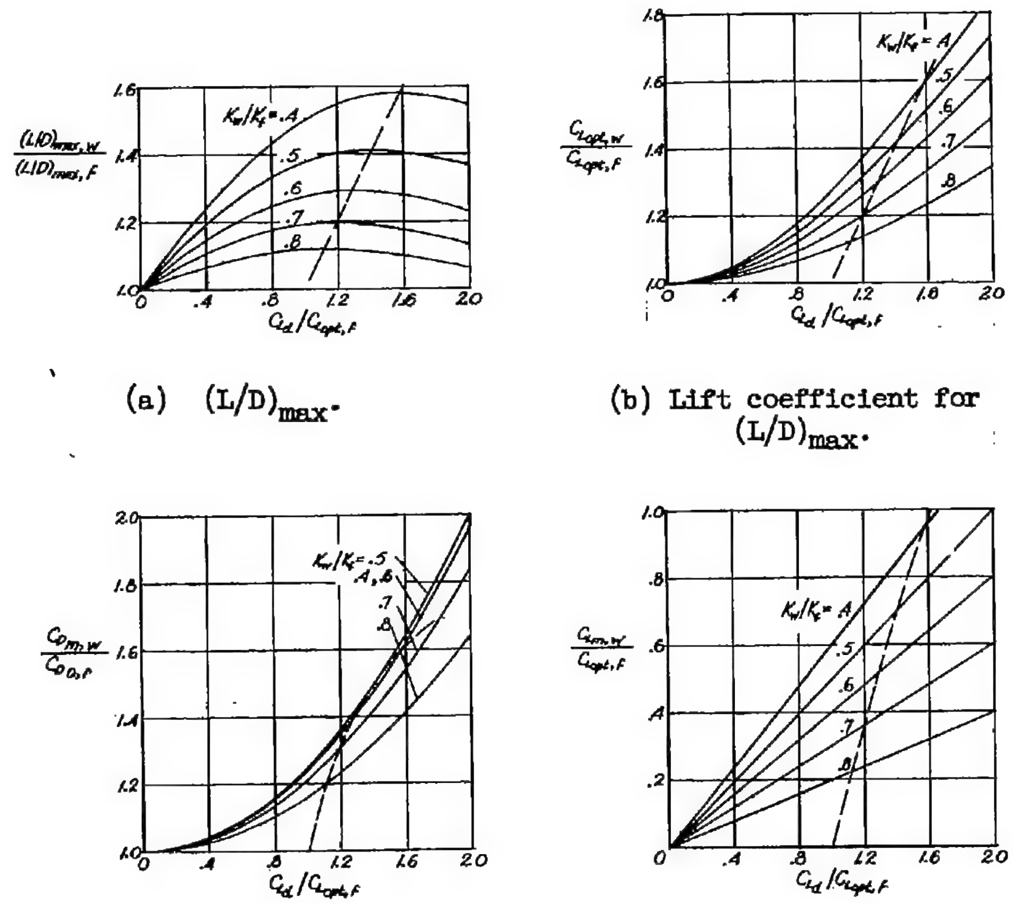


Figure 2.- Basic relations.



(a) $(L/D)_{max}$.

(b) Lift coefficient for $(L/D)_{max}$.

(c) Minimum drag coefficient.

(d) Lift coefficient for minimum drag.

Figure 3.- Variation of several aerodynamic characteristics of minimum-drag wings with design lift coefficient. Dashed lines refer to condition of $\left[\frac{(L/D)_{max,w}}{(L/D)_{max,f}}\right]^{*}$.

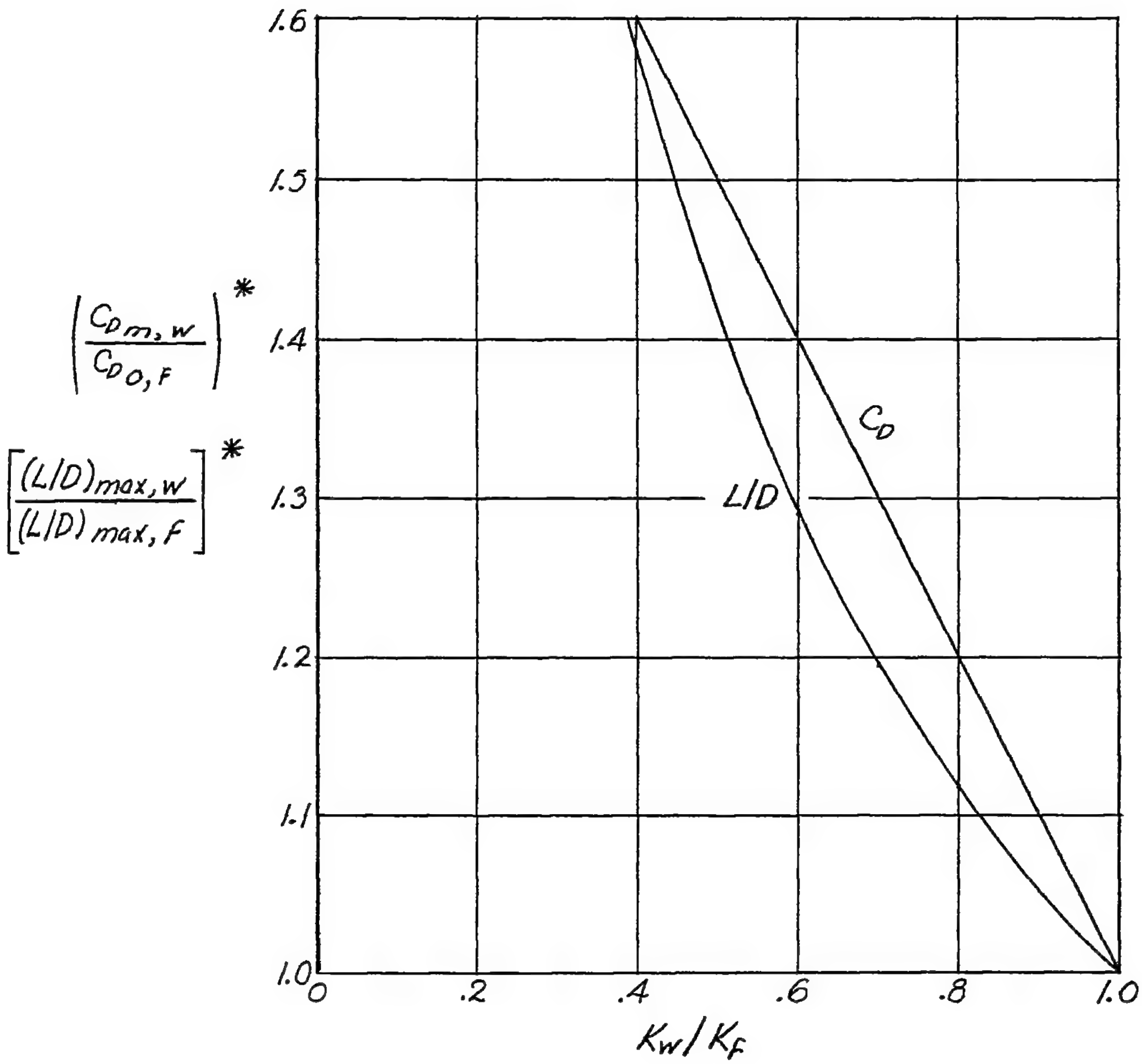


Figure 4.- Maximum values of $\frac{(L/D)_{max,w}}{(L/D)_{max,f}}$ and corresponding values of $C_{D_{m,w}}/C_{D_{0,f}}$.

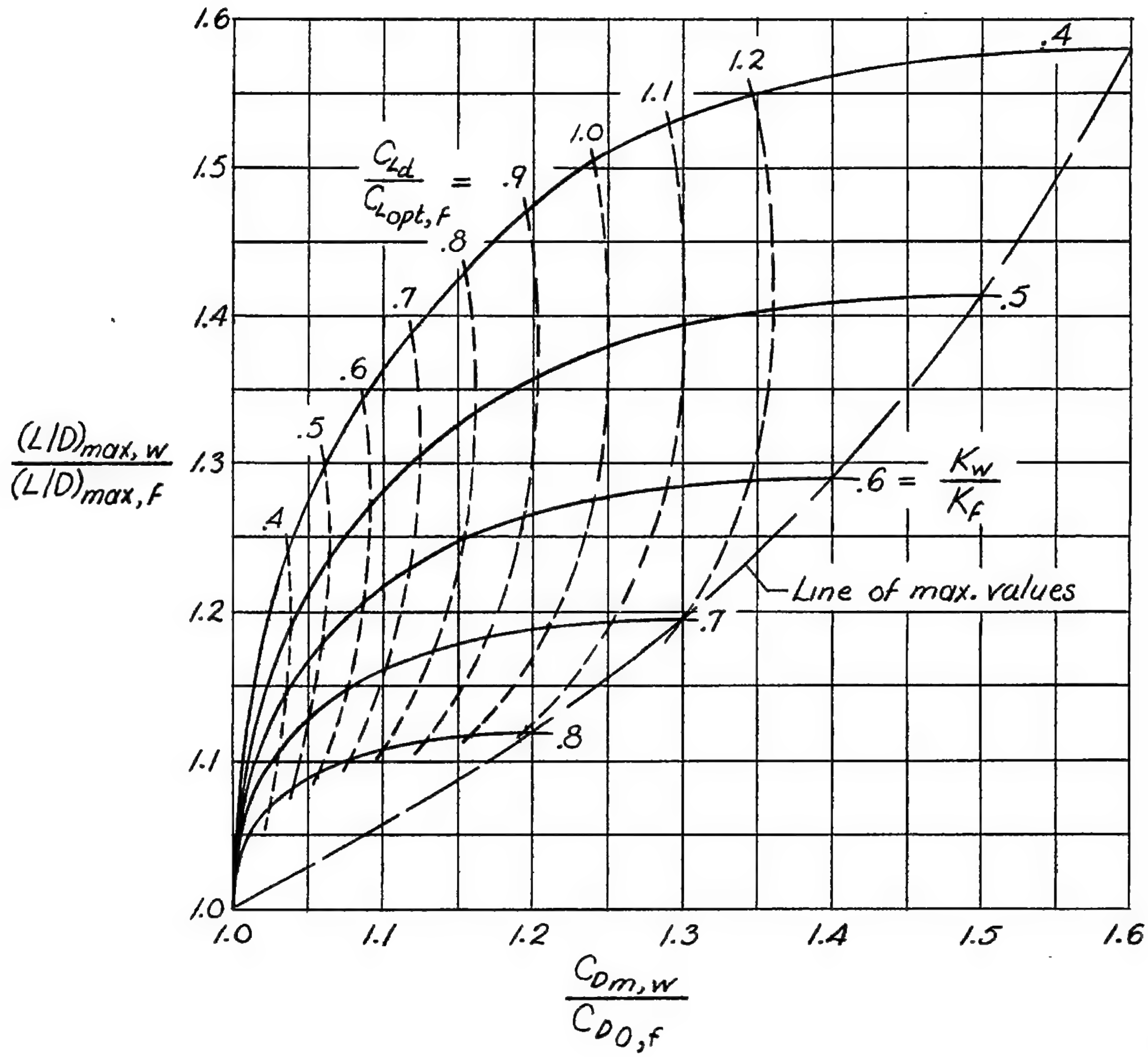
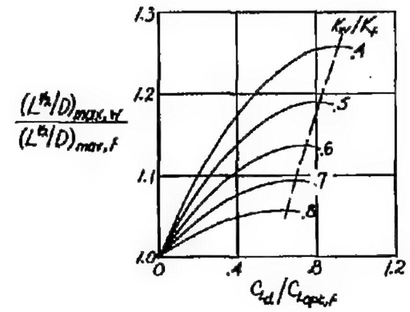
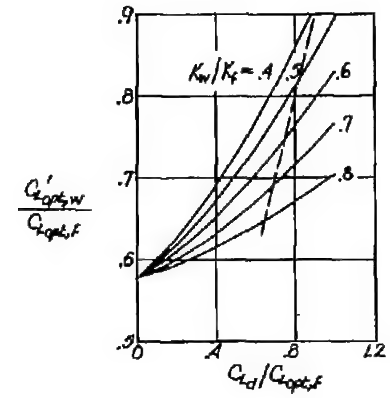


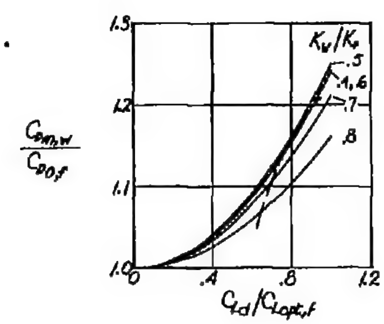
Figure 5.- Variation of $(L/D)_{max}$ ratio with C_D ratio.



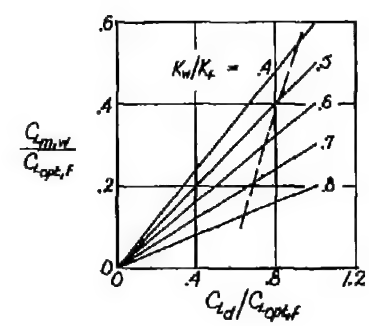
(a) $\left(L^{1/2}/D\right)_{max}$.



(b) Lift coefficient for $\left(L^{1/2}/D\right)_{max}$.



(c) Minimum drag coefficient.



(d) Lift coefficient for minimum drag.

Figure 6.- Variation of several aerodynamic characteristics of minimum-drag wings with design lift coefficient. Dashed lines refer to condition of $\left[\frac{\left(L^{1/2}/D\right)_{max,w}}{\left(L^{1/2}/D\right)_{max,f}}\right]^{**}$.

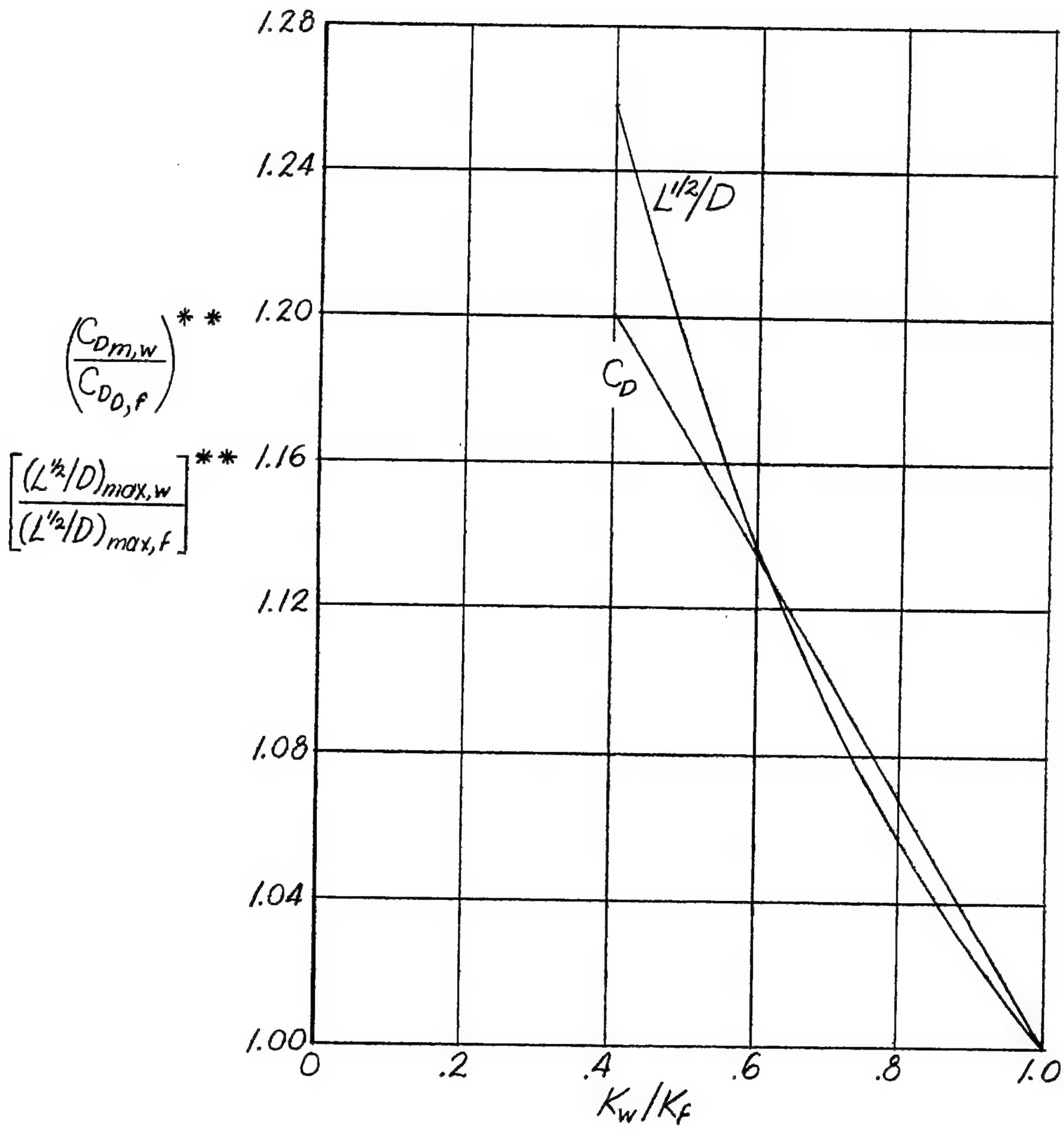


Figure 7.- Maximum values of $\frac{\left(L^{1/2}/D\right)_{max,w}}{\left(L^{1/2}/D\right)_{max,f}}$ and corresponding values of $C_{D_{m,w}}/C_{D_{0,f}}$.

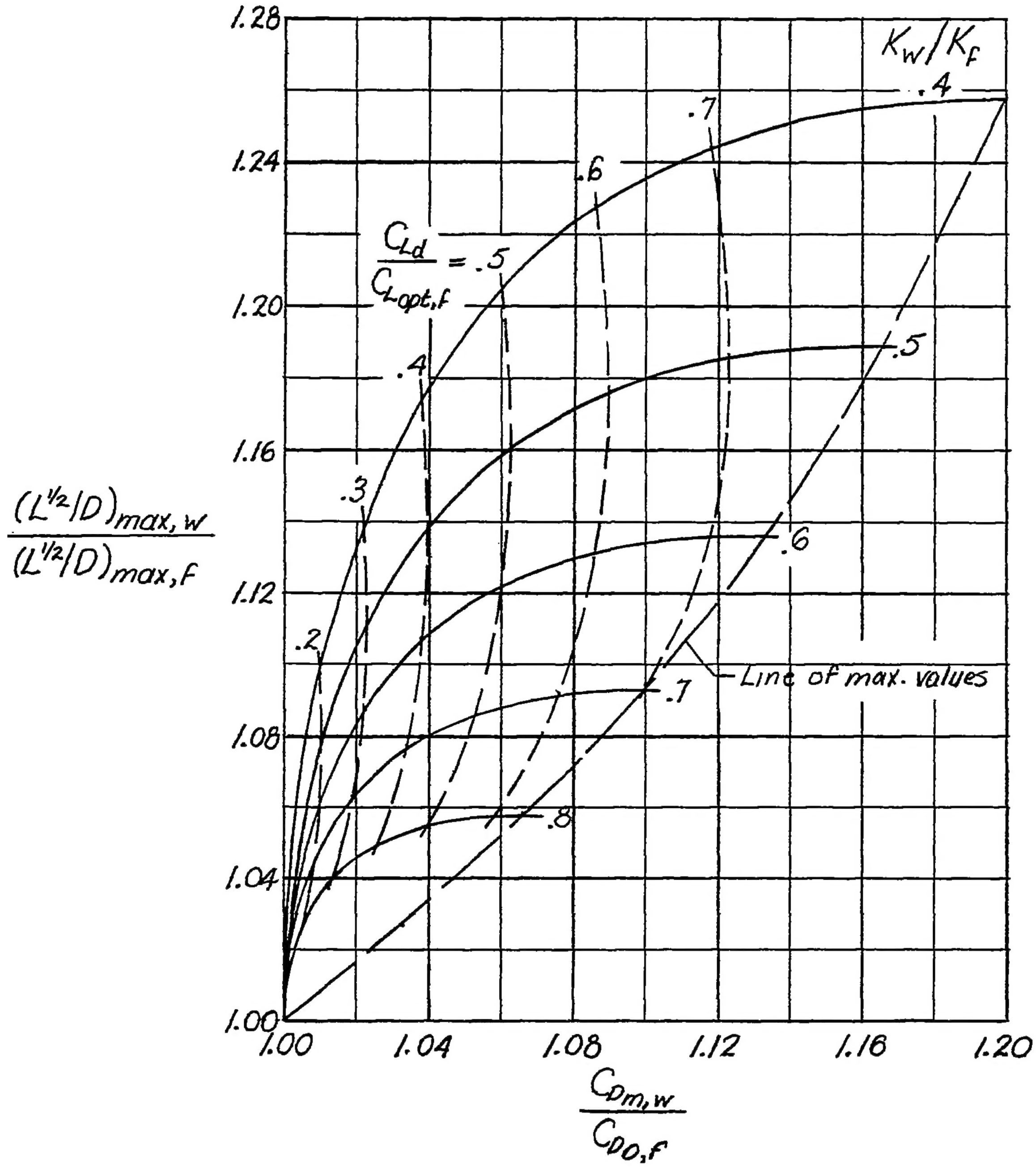
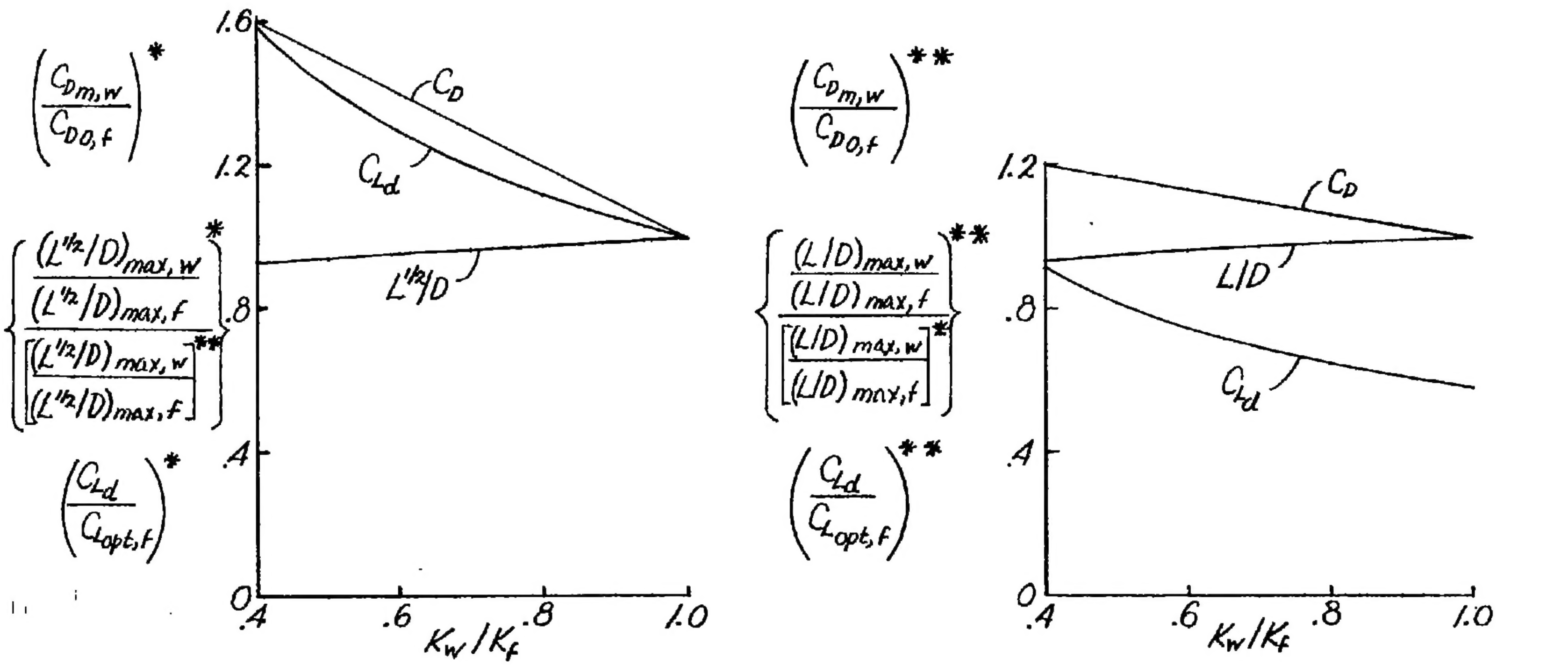


Figure 8.- Variation of $\left(L^{1/2}/D\right)_{max}$ ratio with C_D ratio.



(a) Design lift coefficient chosen to give maximum possible increase in $(L/D)_{max}$.

(b) Design lift coefficient chosen to give maximum possible increase in $\left(L^{1/2}/D\right)_{max}$.

Figure 9.- Comparison of increases in $(L/D)_{max}$ and $\left(L^{1/2}/D\right)_{max}$, and corresponding increases in C_{D_m}, for two design conditions.

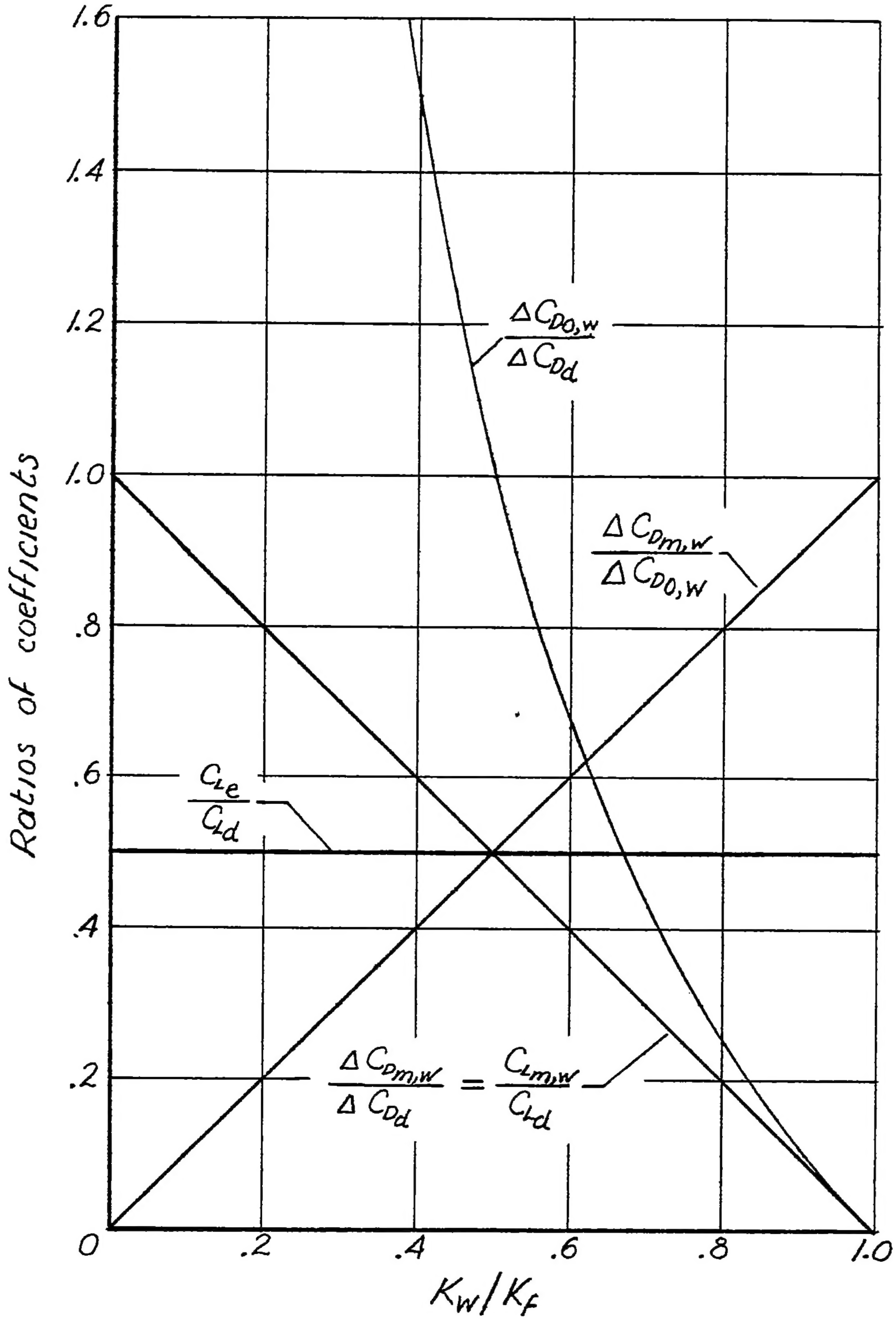


Figure 10.- Relations among characteristics of minimum-drag wings (eqs. (A1) and (A3)).

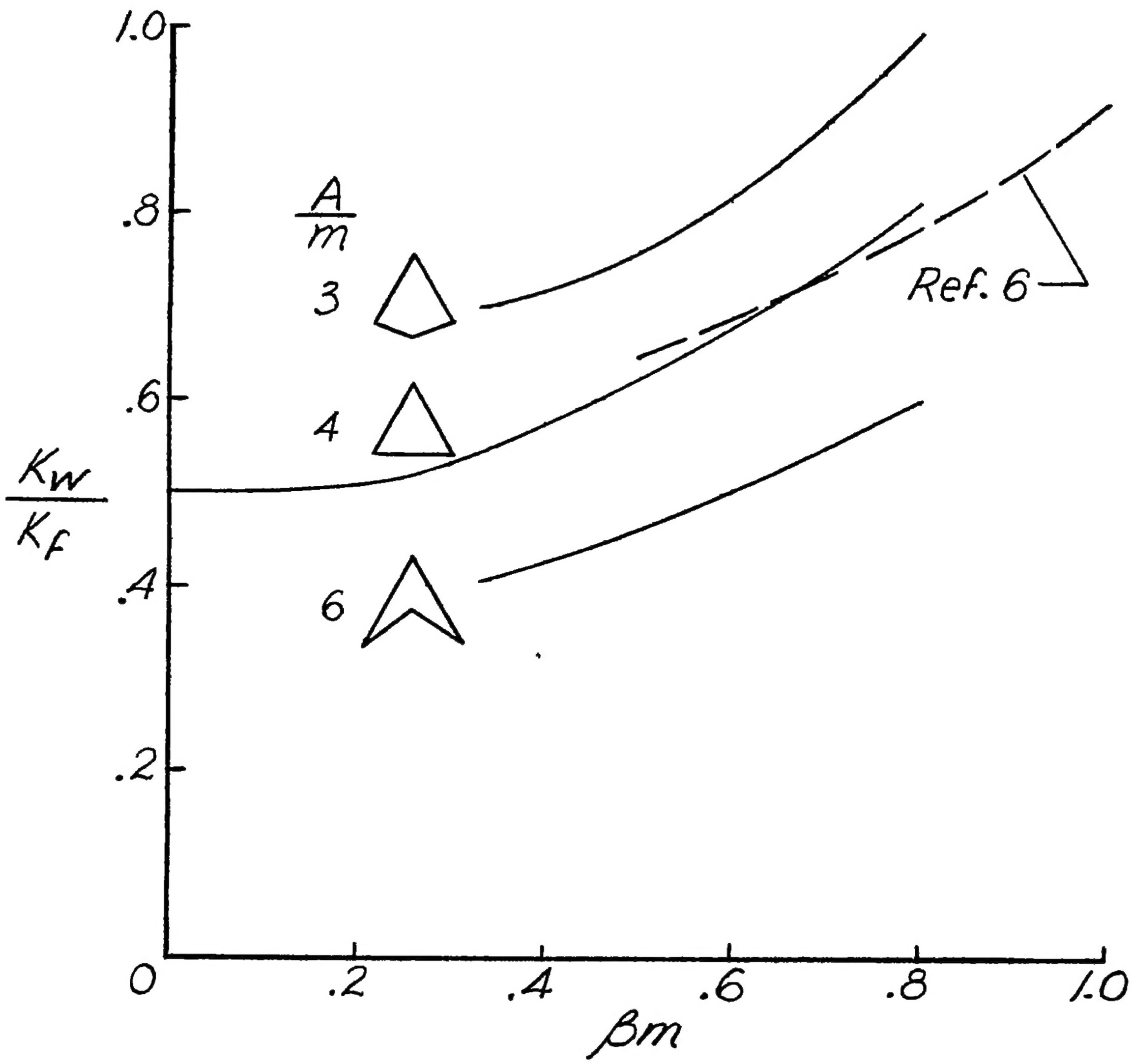


Figure 11.- Values of K_W/K_f for wings with zero tip chord. No leading-edge thrust.